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On New Root Finding Algorithms for Solving Nonlinear Transcendental Equations

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***Abstract*— *In this paper, we present new iterative algorithms to find a root of the given nonlinear transcendental equations. In the proposed algorithms, we use nonlinear Taylor's polynomial interpolation and a modified error correction term with a fixed-point concept. We also investigated for possible extension of the higher order iterative algorithms in single variable to higher dimension. Several numerical examples are presented to illustrate the proposed algorithms.*

Keywords*— *Fixed-point, Error-term, Iterative root finding algorithms, Taylor's series, Transcendental equations.

I. INTRODUCTION

Solving nonlinear transcendental equations is one of the important research areas in numerical analysis, and finding the exact solutions of nonlinear transcendental equations is a difficult task. In several occasions, it may not be simple to get exact solutions. Even solutions of equations exist; they may not be real rational. In this case, we need to find its rational/decimal approximation. Hence, numerical methods are helpful to find approximate solutions. Numerical iteration methods for solving nonlinear transcendental or algebraic equations are useful in applied sciences and engineering problem. A nonlinear equation in n variables does not have the linear form $f = a_n x_n + a_{n-1}x_{n-1} + \cdots + a_1 x_1$, where $a_n, a_{n-1}, \ldots, a_1$ are constants, and also it does not satisfy the conditions such as $f(ab) = f(a)f(b)$ or $f(a+b) = f(a) + f(b)$. For example, the transcendental equation $z = xe^x$ whose solution is given by the *Lambert W* function, W(z), defined as $W(z)e^{W(z)} = z$, which is a fixed point equation of the form $p(x) = x$. This model equation has several applications in science; Physics, Biology, Material science, Quantum statistics, differential equations, signal processing, in the theory of zeta functions etc. [1]. Very especial examples of algebraic nonlinear equations are the polynomials $P_n(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$. Solving science and engineering problems is, directly or indirectly, a root finding. In general, solving transcendental equations is a more difficult when compared with algebraic polynomials, for example, the polynomial solving problem can be transformable to linear eigenvalue problem solving. The complexity of the polynomial problem depends on their degrees and number of terms.

A root-finding problem is to solve for a root r also called a zero or a solution of the equation $f(x) = 0$, such that $f(r) = 0$. The root finding methods are available in the different forms, for example, the methods can be in the form of direct/symbolic, graphical or numerical iterative. Solving transcendental equations for multiple roots is more difficult than solving for a simple root using iterative methods. The available iterative methods/ algorithms are derived from interpolations (linear, quadratic, cubic), perturbation method, variational technique, fixed-point methods, series estimations and so many others. The Taylor's interpolation technique has also many applications; solving differential equations, function estimations and computing, developing and analysis of numerical schemes, etc. Taylor's method is a foundation for calculus of finite differences.

Given any equation can be rewritten as an equivalent equation $g(x) = p(x)$ with the same root *r*. The iteration method to estimate the root *r* is then $g(x_{n+1}) = p(x_n)$. The iterative method converges when $|p'(x_0)/g'(x_0)| \le 1$, with an appropriate initial guess x_0 [2]. In the literature, there exist several root solving methods such as Bisection

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method, Secant method, Regula Falsi, Newton's method and its variants methods to accelerate convergence, namely Chebyshev's method, Halley's method, Super Halley's method etc. [3-12]. These methods may be classified as fixed (single) point methods or methods based on choice of intervals. Nevertheless, choices of initial guesses, interval selections, and existence of derivatives, acceleration convergence, efficiency and possible extensions to two-dimensions or three-dimensions are some common drawbacks in connection with algorithmic complexities [6-24]. In the present paper, we apply Taylor's series approximation of a nonlinear equation $f(x)$ combined with fixed-point concept and some modified error correction term to derive new iterative methods. The concept of function estimation is applied. The function estimation concept is an extension, improvement and a modification of works in [21], see also [22, 23]. Extensions of the proposed algorithms to higher dimensions are also considered to solve the given nonlinear systems. Various interpolation techniques and root-finding algorithms are available in [25-33].

The paper is organized as follows: Section 2 presents the proposed algorithms; in Section 3, we discuss the convergence analysis of the proposed algorithms; in Section 4, we present numerical example to illustrate the algorithms also presented comparisons with other existing methods. Finally, Section 5 presents conclusion and recommendations.

II. PROPOSED NEW ITERATIVE ALGORITHMS

In this section, we derive fixed-point iterative functions of the form $x=\Psi(x)$ from which iterative models $x_{n+1}=\Phi(x_n)$ are obtained.

Definition 1: A point p is said to be a *fixed-point* of $f(x)$ if and only if $f(p)=p$.

The stipulation of the fixed-point concept plays very important role in root finding algorithms. For every root r of $f(x)$, there exists an auxiliary equation $p(x)$ such that $f(r)=r-p(r)=0$. That is r must be a fixed-point of $p(x)$. Solving for a root of $f(x)=0$ by finding a fixed-point of $x=p(x)$ guarantees an iterative method $x_{n+1}=P(x_n)$, n = 0,1,2,…. In literature, one can find several methods that having fixed-point forms. For examples, Newton's method, Halley's method, Chebyshev's method (see equations (3), (10), (11)) etc.

Consider the Taylor's polynomial interpolation of a nonlinear function $f(x)$ differentiable about a number x_0 as:

$$f(x)=f(x_0)+f'(x_0)\frac{(x-x_0)}{1!}+f''(x_0)\frac{(x-x_0)^2}{2!}+\cdots+f^{(n)}(x_0)\frac{(x-x_0)^n}{n!}+\cdots$$

with x is a root r of $f(x)$, very small error $|\Delta x_i|=h$, where $r-x_o=h=\Delta x_i$. Now

$$f(x_0+\Delta x_i)=f(x_0)+\Delta x_i f'(x_0)+0.5\Delta x_i^2 f''(x_0)+\cdots \quad (1)$$

If $f=f(x,y)$, then the Taylor's series expansion in two dimension is

$$f(x+h,y+k)=f(x,y)+(hf_x+kf_y)+\frac{1}{2}(h^2f_{xx}+2hkf_{xy}+k^2f_{yy})+\cdots \quad (2)$$

and if $f=f(x,y,z)$ in 3-dimension, then

$$f(x+h,y+k,z+l)=f(x,y,z)+(hf_x+kf_y+lf_z)+\frac{1}{2}(h^2f_{xx}+2hkf_{xy}+2hlf_{xz}+2klf_{yz}+k^2f_{yy}+l^2f_{zz})+\cdots$$

where $h=\Delta x, k=\Delta y, l=\Delta z$ are small step sizes in x, y, z respectively. The linear estimate from (1), $f(x_o)+\Delta x f'(x_o)\approx 0$ gives the Newton's method for simple roots [18].

$$x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}, \qquad k=0,1,\ldots,n. \quad (3)$$

From a modified Newton's method of order two for multiple roots [16], one has a second order convergent method for finding simple roots [22].

$$x_{k+1}=x_k-\frac{f(x_k)}{U}; \qquad k=0,1,\ldots,n, \quad (4)$$

where $U=f'(x_k)-f(x_k)$ and $h=-\frac{f}{U+f}$ is error/correction part of (3); whereas $h'=-\frac{f}{U}$ is the error term of (4). These error terms play important role to investigate the proposed methods.

(i) A combination of (3) and (4) gives higher order method as a fixed-point formula

$$\Phi(x)=x-\frac{Af(x)}{U+f}+\frac{Bf(x)}{U}, \quad (5)$$

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(ii) If we choose the quadratic model of (1), then $f(x_o+h)=f(x_o)+hf'(x_o)+\frac{h^2 f''(x_o)}{2}\approx 0$. We obtain

$$h=\frac{-2f(x_0)}{2f'(x_0)+hf''(x_0)} \text{ and}$$

$$h=-\frac{f(x_0)+0.5h^2 f''(x_0)}{f'(x_0)} \tag{6}$$

Using the error term $h=\frac{f}{U}$ from (4) in (6), one can get two iterative methods as follows

$$x_{k+1}=x_k-\frac{2f(x_k)[U(x_k)]}{2f'(x_k)[U(x_k)]-f(x_k)f''(x_k)}. \tag{7}$$

$$x_{k+1}=x_k-\frac{f(x_k)}{U(x_k)+f(x_k)}-0.5\frac{f(x_k)^2 f''(x_k)}{f'(x_k)[U(x_k)]^2}. \tag{8}$$

We have, at a simple root, $f'\approx U$. Hence

$$x_{k+1}=x_k-\frac{f(x_k)}{U(x_k)+f(x_k)}-0.5\frac{f(x_k)^2 f''(x_k)}{[U(x_k)]^3}. \tag{9}$$

Inserting the usual error term $h=-\frac{f}{f'}$ from (3) in (6), one has Halley's method (10) and Chebyshev's method (11)

$$x_{k+1}=x_k-\frac{f(x_k)f'(x_k)}{[f'(x_k)]^2-1/2f(x_k)f''(x_k)}. \tag{10}$$

$$x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)}-\frac{1}{2}\frac{[f(x_k)]^2 f''(x_k)}{[f'(x_k)]^3} \tag{11}$$

Also we have the extension of Newton method (12) and Euler method (14), See [5, 21].

$$\varphi(x)=x-\left[\frac{f'(x)}{f''(x)}\pm\frac{\sqrt{[f'(x)]^2-2f(x)f''(x)}}{f''(x)}\right] \tag{12}$$

$$\left.\begin{aligned}\phi(x)&=x-\frac{2H(x)}{1+\sqrt{1-2p(x)}},\\ Q(x)&=\frac{H(x)f''(x)}{f'(x)},\\ H(x)&=\frac{f(x)}{U+f}.\end{aligned}\right\} \tag{13}$$

(iii) Using $U=f'(x)-f(x)$, from the linear estimation of $f(x)$ in (1), one obtains

$$\Omega(x_k)=x_k-\left[\frac{f(x_k)^3+f(x_k)U^2}{U(f(x_k)^2+f'(x_k)U)}\right] \tag{14}$$

For $k=0,1,\dots,n$,

$$\begin{aligned}x_{k+1}&=x_k-\frac{f(x_k)}{U(x_k)+f(x_k)}\frac{f(x_k)+f'^2(x_k)}{f'^2(x_k)+f^2(x_k)};\\ &=x_k-\frac{f(x_k)+(u(x_k)+f(x_k))^2}{(u(x_k)+f(x_k))^2}\\ &\times\left(\frac{f(x_k)}{u(x_k)+f(x_k)}-\left(\frac{f(x_k)}{u(x_k)+f(x_k)}\right)^3\right)\end{aligned} \tag{15}$$

(iv) Suppose consider the cubic model of $f(x)$ as

$$\begin{aligned}f(x_o+h)&=f(x_o)+hf'(x_o)\\ &+\frac{1}{2}h^2 f''(x_o)+\frac{1}{6}h^3 f'''(x_o)\approx 0.\end{aligned} \tag{16}$$

$$(16)\Rightarrow hf'+\frac{1}{6}h^3 f'''=h(f'+1/6h^2 f''')=-(f+1/2h^2 f'')$$

Now we have $h=-\frac{6f+3h^2 f''}{6f'+h^2 f'''}$. Inserting a correction term $h=\frac{f}{U}$, we have

$$h=-3\frac{2ff'^2-4f'f^2+2f^3+f^2f''}{6f'^3-12f'^2f+6f'f^2+f^2f'''}$$

This gives an iteration function using (i), (ii), (iii).

$$\psi(x)=x-3\frac{2ff'^2-4f'f^2+2f^3+f^2f''}{6f'^3-12f'^2f+6f'f^2+f^2f'''} \tag{18}$$

Replacing the f''' by $\frac{f'f''}{f}$, we get the method

$$\psi(x)=x-3\frac{2ff'^2-4f'f^2+2f^3+f^2f''}{6f'^3-12f'^2f+6f'f^2+ff'f''} \tag{19}$$

(v) Now, we derive the new iterative functions using Taylor expansion and iterative methods are given (22), (23), (24), (25) and (26).

Consider

$$f(x_o)+hf'(x_o)+\frac{1}{2}h^2 f''(x_o)+\frac{1}{6}h^3 f'''(x_o)\approx 0, \tag{20}$$

which gives

$$hf'(x_0)+\frac{1}{2}h^2 f''(x_0)=-\left[f(x_0)+\frac{1}{6}/6h^3 f'''(x_0)\right], \text{ and}$$

hence

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$$h = -\frac{6f(x_0)+h^3 f'''(x_0)}{6f'(x_0)+3hf''(x_0)}. \tag{21}$$

If we substitute $h=\frac{f}{U}$ in (21), we get

$$x_{k+1} = x_k - 2\frac{U[f(x_k) - 1/6U^3 f'''(x_k)}{2f'(x_k)U - f(x_k)f''(x_k)} \tag{22}$$

$$x_{k+1} = x_k - 2(f'(x_k) - f(x_k)) \times\left[f(x_k) - \frac{1}{6}\left(\frac{f(x_k)}{f'(x_k)-f(x_k)}\right)^3 f'''(x_k)\right] / [2f'(x_k)(f'(x_k)-f(x_k)) - f(x_k)f''(x_k)] \tag{23}$$

Utilizing $f''' = \frac{f'f''}{f}$ in (23), we have

$$x_{k+1} = x_k - 2(f'(x_k) - f(x_k)) \times\left[f(x_k) - \frac{1}{6}\frac{f^2(x_k)}{(f'(x_k)-f(x_k))^3} f'(x_k)f''(x_k)\right]. / [2f'(x_k)(f'(x_k)-f(x_k)) - f(x_k)f''(x_k)] \tag{24}$$

And if $f''' = \frac{f''^2}{f}$, we obtain

$$x_{k+1} = x_k - 2(f'(x_k) - f(x_k)) \times\left[f(x_k) - \frac{1}{6}\frac{f^2(x_k)}{(f'(x_k)-f(x_k))^3} f''(x_k)^2\right] / [2f'(x_k)(f'(x_k)-f(x_k)) - f(x_k)f''(x_k)] \tag{25}$$

One may also obtain

$$x_{k+1} = x_k - 2\frac{UVf''(x_k)}{2f'(x_k)U - f(x_k)f''(x_k)}, \tag{26}$$

where $U = f'(x_k) - f(x_k)$ and

$$V = f(x_k) - \frac{1}{6}\left[\frac{f(x_k)}{U}\right]^3.$$

Note: (Nonlinear Systems in Two-dimensional Space) The extension of the methods in (7) and (8) can be expressed as follows. The equation (7) can be written in two-dimensional space as $x_{k+1} = x_k - (N)[1-W]^{-1}$, where $N = x_k - \varphi(x_k)$, and $W = 0.5ff''(f')^{-1}[f'-f]^{-1}$; and the equation (8) as $x_{k+1} = (x_k - \phi(x))[1-0.5ff''((f'-f)^{-1})^2]$. Also (9) is expressed as $x_{k+1} = x_k - (N)[1-W(f')^3]$. Where $\varphi(x)$ is Newton's iteration function to solve $f(x)=0$ in one variable. The Newton's method to solve a system of equations $F(X)=0$ is given by $\Delta X_k = -J^{-1}(X_k)F(X_K)$. Where J is the Jacobean of F and J^{-1} is its inverse when exists.

III. CONVERGENCE ANALYSIS

In this section, we present the convergence analysis of the proposed methods. The basic concepts and definitions can be found in [2, 4, 5, 18] for further details.

Definition 2: Let (x_n) be the sequence of approximate roots obtained using the iterative method $x_{n+1} = p(x_n)$ for exact root r. The *convergence is said to be reached* if $\lim p(x_n) = r$, or equivalently $\lim (e_n) = 0$. The sequence of errors is denoted by $e_n = r - p(x_n)$.

Theorem 1: (Fixed-Point Iteration Theorem) Let $f(x)=0$ be written in the form $x = p(x)$. Assume that $p(x)$ has the following properties:

- For all x in $[a, b]$, $p(x)$ is in $[a, b]$, i.e., $p(x)$ takes every value between a and b.
- $p'(x)$ exists on (a, b) with the property that there exists a positive constant $k < 1$ such that $|p'(x)| < k$ for all x in (a, b).

Then

(i) there is a unique fixed point p of $p(x)$ in $[a, b]$.

(ii) for any x_0 in $[a, b]$, the sequence $x_{n+1} = p(x_n)$, n = 0, 1, 2,… converges to the fixed the point p, i.e., to the root $r = p$ of $f(x)=0$.

Theorem 2: (Order of Convergence) Assume that $\phi(x)$ has sufficiently many derivatives at a root r of $f(x)$. The order of any one-point iteration function $\phi(x)$ is a positive integer p, more especially $\phi(x)$ has order p if and only if $\phi(r) = r$ and $\phi^{(j)}(r) = 0$ for $0 < j < p$, $\phi^{(p)}(r) \neq 0$.

Now we can prove the order of convergence of algorithm in (7) as follows. Consider the iterative method $x_{k+1} = x_k - (N)[1-M]^{-1}$, or

$$x_{k+1} = x_k - (x_k - \varphi(x_k))[1-0.5ff''f'^{-1}[U]^{-1}]^{-1}, \tag{27}$$

where $N = x_k - \varphi(x_k), M = 0.5ff''f'^{-1}[U]^{-1}$. We can write iteration equation (27) as $\phi(x) = x - (x-\varphi(x))w$, where $w(x) = [1-M[U]^{-1}]^{-1}$ and $x = \varphi(x)$, Newton's iteration function.

Let r be a simple root of $f(x)=0$. We have $\varphi(r) = r$, $\varphi(x) = x$ and $\phi(x) = x$. And $\varphi'(r) = 0$

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but $\varphi''(r) \neq 0$. Differentiating $\phi(x)$, where $\phi(x) = x - (x - \varphi(x))w$, we find that $\phi'(r) = \phi''(r) = 0$ but $\phi'''(r) \neq 0$. Therefore, $p \geq 3$.

Conversely, if p = 3, then we can show that $\phi'(r) = \phi''(r) = 0$ but $\phi'''(r) \neq 0$. Hence, (7) or (27) is third order convergent method. Similarly, we can do for the other methods.

IV. NUMERICAL EXAMPLE

Consider the following equations to test the efficiency of the proposed methods.

(1) $f_1(x) = 5/2x^3 - 5/2x - 5/2 = 0$, with x_0 = 1, 2

and root $r \approx$ 1.324718 in (1, 2),

(2) $f_2(x) = 6x - 2\cos x - 2 = 0$, x_0 = 0, 1, 2,

$r \approx$ 0.607102 in (0, 1).

(3) $f_3(x) = \cos x + x^3 - e^x = 0$, x_0 =-2.5, -1, -0.5,

$r \approx$ -0.649565 in (-1, 0).

(4) $f_4(x) = 4x^6 - 4x - 4 = 0$, x_0 = 1, 2,

$r \approx$ 1.134724 in (1, 2)

(5) $f_5(x) = \log_{10}(x) - 2x + 2$, x_0 = 0.5, 1.5, 2.2,

r =1.000000 in [1, 2) [5].

The comparisons of the results obtained by Newton method (NM), Chebyshev's method (CM), Halley's method (HM) with the proposed methods in equations (4), (7), (8), (23), and (24) are given in Table 1. The implementation of the proposed algorithms in C++ is done, and the number of iterations taken to converge to a root r with tolerance Tol = 10^{-7} is recorded. The "Functions (f)" in table refers to the number of functional evaluations up to derivatives and "Nar" denotes average number of iterations. The efficiency indices can be calculated as $e = f^{1/p}$, where p is order of convergence, for example, if f = p = 2, then e = 1.4142, also e = 1.4142 for p = f = 4. Hence, the higher order doesn't give guarantee for the faster convergent or better efficient.

Table 1: Numerical Results of Convergence Analysis

f, x_0	Nm	(4)	CM	(7)	(8)	HM	(23)	(24)	(25)
f_1: 1, 2	5, 5	3, 3	4, 3	2, 2	2, 4	3,3	2, 3	3, 3	3,2
f_2 :0, 1, 2	3,3, 3	5, 5, -	3, 3, 3	2, 3, 3	2, 3, 3	2,2,3	3, 2, 3	3, 2, 3	3,2,3
f_3 :-2.5,-1,-.5	7,5, 4	6, 5, 4	4, 3, 4	4, 3, 3	4, 3, 3	4,3,3	4, 3, 3	4, 3, 3	4,3,3
f_4 ,1,2	5, 7	3, 5	3, 4	2, 2	2, 2	3, 4	2, 3	2, 3	3,2
f_5 :.5,1.5,2.2	3,3, 3	4, 5, 6	3, 2, 3	3, 2, 3	3, 2, 2	3, 2,3	2, 2, 3	2, 2, 3	2,2,3
Functions (f)	2	2	3	3	3	3	4	3	3
Nar	4	4	3	3	3	3	3	3	3

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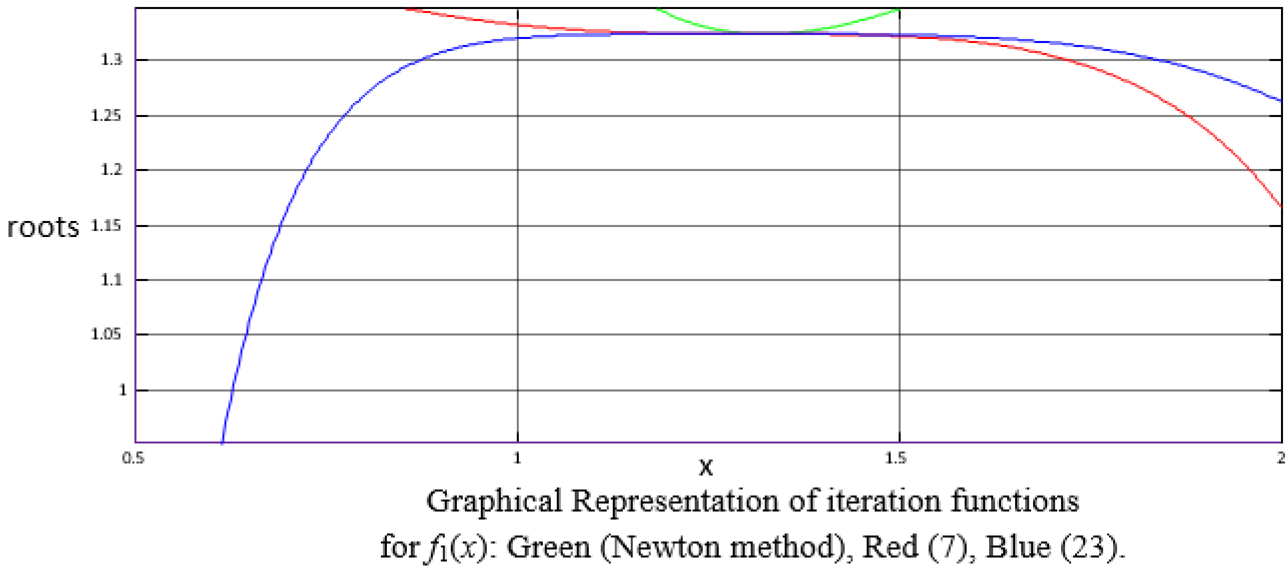

Fig.1: Graphs of three fixed point methods

V. DISCUSSIONS

In Table 1, the average number of iterations elapsed to converge to the root *r* and numbers of function evaluations up to derivatives have been determined. The graphs of the fixed-point functions for some methods are also shown. The use of the error/correction term results in fast convergent methods, but may not affect the number of functional evaluations up to derivatives. Moreover, it can be seen from the graphs that the new methods converge at single fixed-point (root *r*) even for larger intervals containing the root *r* when compared with existing method. It is also found that higher order does not mean better efficient. Efficiency *e* depends on number of function evaluations *f* and order *p*. In addition to new methods, the work has improved, in terms of proofs, graphs and results, by few authors. In this study, we also reviewed several applications of root finding algorithms in science and engineering. For example, (1) to find intersection of curves, (2) computing the critical or stability points of functions, (3) to determine extreme values of optimization problems, (4) for solving matrix eigenvalue problems and applicable in other designing problems.

VI. CONCLUSIONS AND RECOMMENDATIONS

In the paper, we applied Taylor's nonlinear polynomial interpolation and a correction term with fixed-point concept to obtain new iterative methods for estimating simple roots of nonlinear transcendental or algebraic equations. The correction technique applied to both quadratic and cubic models does affect convergence. We have shown possible extensions for solving higher dimensional nonlinear systems. In our future work, we shall present further analyses of these algorithms and other higher order iterative algorithms with applications.

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